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Level Set Method for Image Segmentation and Bias Field Estimation using Coefficient of Variation with Local Statistical Information

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Abstract— Most of the image processing techniques use image regional information for image segmentation, image registration, etc. Region based methods for image segmentation with bias field estimation based on the statistical information of different region such as intensity mean, intensity distribution etc. These methods rely on the assumption of spatial invariance in the image domain that is not always valid for most images. We present a new variational model in this paper for segmenting important tissues in the image with the estimate of bias field. We use a term like coefficient of variation as a criterion function based on the local statistical information, and define energy functional in the level set framework. Coefficient of variation is a relative measure uses relative intensity information in the neighborhood for achieving the desired result. Minimization of this energy functional gives estimate of bias field and segment important tissue in the image domain. Empirical results reveal that our new proposed model is independent of initialization and accurate in contrast with the existing model.

Index Terms—Image segmentation, Intensity inhomogeneity, Coefficient of variation (CoV), Bias estimation, level sets, MRI, Functional minimization.

1. Introduction

MRI is a powerful imaging techniques in medical images used for the visualization of inner body structure, while bias field is unwanted smooth signal that occur in most MRI due to intensity inhomogeneity in their magnetic field. This undesirable smooth signal modifies high frequency information of the image, which create many problems in image processing algorithms such as image segmentation, image registration, image classification etc. In the presence of bias field it is difficult to segment important tissue in the image domain due to intensity overlapping in different region. Most of the image segmentation algorithms [1],[2] rely on the assumption of spatial invariance, which are not applicable to images with bias field. Level set methods as a numerical techniques used for the evolution of contour by embedding the contour in one higher dimension is called level set function. This method is design for a problem to move the contour front forward in some direction if the level set function is positive otherwise backward to reach the desired object boundary. Active contour methods commonly use two approaches; Edge based models [4],[5] and Region based models [1],[2],[3] for classifying pixel value belonging to object or region. Edge-based models uses important feature in the image domain for segmentation such as edges that separate regions. These models even works in the presence of bias field, but it is hard to find edges of different regions in noisy images. Region-based models are superior to edge-based model, because it uses regional information for image segmentation that take more pixel value than edge and will have more information for the movement of active contour. Region-based models rely on the assumption of spatial invariance; example is Chan and Vese model [1]. This model does not work for the images with heterogeneous regions. Further Chan and Vese generalized his work [3] into multiphase level set formulation by using multiphase level set functions for different regions such models are called piecewise smooth (PS). These models do not base on the assumption of intensity homogeneity and work well even in the presence of intensity inhomogeneity. However, it is difficult to compute numerically.

2 BACKGROUND REVIEW

2.1 Mumford and Shah Functional Model

Among variational models Mumford and Shah model is the oldest model used for image segmentation. Let v and I are the functions define on $\Omega \subset R^2$ .The aim of image segmentation is to find out the desired v from the given image I through a defined model. Mumford and Shah proposed the following variational model [6] gives disjoint region in the image domain

$$E(v,C) = \lambda_1 \int_{\Omega} |v - I|^2 \, dxdy + \lambda_2 \int_{\Omega \setminus C} |\nabla v|^2 \, dxdy + \lambda_3 |C|, \quad (1)$$

where $\lambda_1 > 0, \lambda_2 > 0$ and $\lambda_3 > 0$ are parameters use for the adjustment in the model, and $|C|$ gives the boundary length. Eq.1 has three terms. First term is data fitting term, second term use to regularize the contour, and third is the smoothing term. The minimization of this energy functional through partial differential equation has some problems. The main problem is discrete values on boundaries, which create discontinuities into energy functional. To surmount this problem Chan-Vese [1] used level set methods and solved Mumford and Shah Segmentation Problem.

3 Framework for Image Segmentation with Bias Field Correction

3.1 Image Model:

Image processing techniques for image segmentation uses the image model for segmenting important region in the image domain. It assumed to have two important components, the bias field and the true image. We use multiplicative model for modeling bias field, an observed image *I* can be expressed in term of true image *q*, bias field *b*, and additive noise *n* as

$$I = bq + n, \qquad (2)$$

where *b* is the bias field, q is the true image presumed to be constant in different region and *n* assumed to be zero mean additive Gaussian. Our basic objective is to extract bias field components from the given image and use them to correct intensity non uniformity. We assume that bias field is smooth and slowly varying throughout the domain can be approximated by constant values in the neighbourhood. The true image q will have constant values $c_1, c_2, \ldots c_n$ in disjoint region $\Omega_1, \Omega_2, \ldots \Omega_n$ reflect the physical properties in these regions.

3.2 Local Intensity Clustering Property:

Region based methods use different region descriptor of the intensities, such as intensity mean, intensity distribution, etc to segment important tissue in the image domain. In the presence of bias field it is to define a region descriptor that gives information about different regions in the image domain, because of overlapping between the intensity distributions in $\Omega_1, \Omega_2, \ldots \Omega_n$ regions. So, it is not possible to use intensity information for region segmentation. However, in the formulation of our proposed model we use the local intensity clustering property for achieving the desired result. According to this property at every point y belong to Ω, we define a circular neighborhood with radius *r* is given by

$$\{O_y = x : |x - y| < r\}. \qquad (3)$$

Partition of the full domain Ω also partition of the small circular region O_y i.e. $O_y \cap \bigcap_{i=1}^{N} \Omega_i$ form partition O_y.

The values of the smooth function *b*(x) for $x \in O_y$ are close to *b*(y) i.e. *b(x) ≈ b(y)* for x ∈ O_y. Therefore the intensity value *b(x) q(x)* in subregion $O_y \cap \Omega_i$ are close to *b(y)* c_i for $x \in O_y \cap \Omega_i$ i.e. *b(x)q(x) ≈ b(y)* c_i *for* $x \in O_y \cap \Omega_i$ where *q*(x) represent true value and function of x and *b(y)* c_i are the estimated constant values in different regions. Thus, the observed image in (2) can be rewritten as

$$I = b(y)c_i + n(x), \qquad (4)$$

where n(x) Gaussian noise with zero mean and the intensities values in $O_y \cap \Omega_i$ form a cluster with center m_i =b(y) c_i, may be treated are sample values drawn from Gaussian distribution with mean m_i.

3.3 Proposed Model:

In the neighborhood O_y we use local intensity clustering property to classify intensity values into N cluster with centers m_i =b(y) c_i i=1, 2, 3... N. K-mean clustering algorithm minimizes the clustering criterion function through iterative process [7]. In the proposed model, we use a modified form of K-Mean clustering criterion function to classify the intensity values in the neighborhood O_y into disjoint regions. For the intensities *I*(x) the continuous version of the clustering criterion function can be written as

$$R_y = \sum_{i=1}^{N} \int_{O_y} |I(x) - m_i|^2 \, v_i \, dx \qquad (5)$$

where v_i is the membership function of the region Ω_i, $v_i = 1$ for x ∈ Ω_i and $v_i = 0$ for x ∉ Ω_i, m_i is the ith cluster center. The above energy equation can be rewritten as

$$R_y = \sum_{i=1}^{N} \int_{\Omega_i \cap O_y} |I(x) - m_i|^2 \, dx \qquad (6)$$

The clustering criterion function in (6) is defined in term of centre approximation by m_i =b(y) c_i for classifying the intensities in O_y as

$$R_y = \sum_{i=1}^{N} \int_{\Omega_i \cap O_y} |I(x) - b(y)c_i|^2 \, dx \qquad (7)$$

We introduce a nonnegative function window function K(y-x) is defined as K(y-x)=0 for x ∉ O_y. The energy function in term of center approximation and window function can be rewritten as

$$F_y = \sum_{i=1}^{N} \int_{\Omega_i} K(y - x) |I(x) - b(y)c_i|^2 \, dx \qquad (8)$$

Minimization of the energy functional in Eq.8 results image segmentation with bias field estimation. It does not work well in the images with more intensity Non-uniformity. Therefore, in the proposed model we present a simple modification of the energy functional in Eq.8 dividing the criterion function by the square of the approximated cluster mean b(y) c_i to exploit the relative information of the cluster. The modified form of energy in Eq.8 can be defined as

$$F_y = \sum_{i=1}^{N} \int_{\Omega_i} K(y - x) \left(\frac{|I(x) - b(y)c_i|^2}{(b(y)c_i)^2} \right) dx \qquad (9)$$

Our proposed method uses the modified form of the K-mean clustering criterion function as tool in the energy functional F_y

to classify the intensities in the neighbourhood O_y, which partition O_y such as $O_y \cap \bigcap_{i=1}^{N} \Omega_i$. Smaller the value of the energy functional F_y result good classification. Good classification of the intensity values in the entire domain Ω can be obtained by minimizing F_y for all value of y in Ω. Thus we minimize F_y for all value of y in Ω, which can be obtained minimizing $\int F_y dy$ for all y in Ω such as $F = \int F_y dy$

$$F = \int \left(\sum_{i=1}^{N} \int_{\Omega_i} K(y-x) \left(\frac{|I(x) - b(y)c_i|^2}{(b(y)c_i)^2} \right) dx \right) dy \quad (10)$$

In Eq.10 the integration is over the entire domain, therefore, we omit Ω_i in the subscript. Minimization of the energy functional in (10) with respect to the regions, $\Omega_1, \Omega_2, \ldots \Omega_n$, bias field *b*, and constants $c_1, c_2, \ldots c_n$ segment important tissues in the images with estimate of bias field. In the proposed method we use truncated Gaussian distribution as a kernel function defined by

$$K(v) = \begin{cases} \dfrac{1}{\beta} e^{-\frac{1}{2}(|v|/\sigma)^2} & for\ all\ |v| < r \\ 0 & otherwise \end{cases}$$

where σ is the scale parameter of the Gaussian function, β is the normalization constant and *r* is radius of the neighbourhood O_y. We select *r* for the neighbourhood O_y on the basis of the degree of intensity inhomogeneity. The bias field vary faster in more intensity inhomogeneous regions; therefore, we take small value for the radius *r* in the neighbourhood O_y.

4. Energy Minimization using level set Formulation:

We use level set method that was devised by Osher [7]. In the propose method we use level set functions takes positive and negative signs to represent different region in the image domain. It is difficult to minimize the above energy in the present form; therefore, we use level set formulation and well known variational method for the minimization of energy in (10). Suppose $\phi : \Omega \to R$ be a level set function, its signs partition the domain into two disjoint regions such as Ω_1 = {x: ϕ (x)>0} and Ω_2 = {x: ϕ (x) <0}. It is called two phase level set formulation. To partition the image domain Ω into more than two disjoint regions $\Omega_1, \Omega_2, \ldots \Omega_n$, we use two or more than two level set functions are called multiphase level set formulation.

4.1 Two Phase Level Set Formulation:

To partition the image domain into two regions Ω_1 and Ω_2, a single level set function ϕ is used in the level set formulation. The membership functions such as $D_1(\phi) = H(\phi)$) and $D_2(\phi) = 1 - H(\phi)$ are used to define disjoint regions Ω_1 and Ω_2. Where H is the Heaviside function [8],[9]. Thus, the energy functional in (10) in term of level set formulation can be written as

$$F = \int \left(\sum_{i=1}^{2} \int_{\Omega_i} K(y-x) \left(\frac{|I(x) - b(y)c_i|^2}{(b(y)c_i)^2} \right) D_i(\phi(x)) dx \right) dy \quad (11)$$

By exchanging the order of integration we have,

$$F = \int \left(\sum_{i=1}^{2} \int_{\Omega_i} K(y-x) \left(\frac{|I(x) - b(y)c_i|^2}{(b(y)c_i)^2} \right) dy \right) D_i(\phi(x)) dx \quad (12)$$

The above energy *F* depends on the level set function ϕ, the constants $c_1, c_2, \ldots c_n$, and the bias field *b* can be rewritten as

$$F(\phi, \mathbf{c}, b) = \int \left(\sum_{i=1}^{2} a_i(x) D_i(\phi(x)) dx \right) \quad (13)$$

where a_i is given by

$$a_i(x) = \int K(y-x) \left(\frac{|I(x) - b(y)c_i|^2}{(b(y)c_i)^2} \right) dy. \quad (14)$$

The simple expression for the function a_i is given by

$$a_i(x) = I_k - \frac{2}{c_i} I\left(\frac{1}{b} * K\right) + \frac{I^2}{c_i^2}\left(\frac{1}{b^2} * K\right), \quad (15)$$

where * is the convolution operator, and $I_k = \int K(y-x)$ is one in the image domain Ω except near the boundary. The data fitting term in (13) can be expressed in a variational level set framework is given by

$$R(\phi, b, \mathbf{c}) = F(\phi, b, \mathbf{c}) + \lambda_1 R_q(\phi) + \lambda_2 L(\phi) \quad (16)$$

where $R(\phi)$ is the distance regularize have been used in different variational method [8] for the smooth curve evolution is given by

$$R_q(\phi) = \int q(|\nabla \phi|\, dx \quad (17)$$

and $L(\phi)$ is the arc length of the contour can be defined as

$$L(\phi) = \int |(\nabla H(\phi)|\, dx \quad (18)$$

The minimization of the energy in (16) results image segmentation with bias field estimation and this can be obtained by iterative process. For minimization, we partially differentiate the energy functional $R(\phi, b, \mathbf{c})$ in every iteration with respect to ϕ,*b and c*.

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4.2 Energy Functional Minimization With Respect to ϕ:

The energy functional minimization in (16) with respect to ϕ can be obtained by using the Gateaux derivative $\frac{\partial R}{\partial \phi}$ of the functional $R(\phi, b, \mathbf{c})$, and solve the following gradient flow equation for the steady state solution.

$$\frac{\partial \phi}{\partial t} = -\frac{\partial R}{\partial \phi} \qquad (19)$$

The expression for the Gateaux derivative can be obtained by using calculus of variation [9], hence the gradient flow equation is given by

$$\frac{\partial \phi}{\partial t} = -\delta(\phi)(a_1 - a_2) + \lambda_1 div(d_q(|\nabla\phi|)\nabla\phi) + \lambda_2 \delta(\phi) div\left(\frac{\nabla\phi}{|\nabla\phi|}\right) \qquad (20)$$

Finite difference method can be used to implement the smooth evolution of level set function in (20) and update the constant value (c_1, c_2) in **c** and *b* through minimization of energy functional with respect to **c** and b respectively.

4.3 Energy Functional Minimization With Respect to c:

Minimize the energy functional $R(\phi, b, \mathbf{c})$ with respect to **c** keeping ϕ and b are constant gives the optimal value of **c**=(c_1, c_2) as

$$\hat{c}_i = \frac{\int (\frac{1}{b^2} * K) I v_i dy}{\int (\frac{1}{b} * K) v_i dy} \quad i = 1,2, \; with \; v_i = D_i(\phi(x)) \quad (21)$$

4.4 Energy Functional Minimization With Respect to b:

Minimize the energy functional $R(\phi, b, \mathbf{c})$ with respect to b keeping ϕ and c are constant gives the optimal value of b as.

$$\hat{b} = \frac{(W^{(2)} I * K)}{W^{(1)} * K}, \qquad (22)$$

Where,

$$W^{(1)} = \sum_{i=1}^{2} \frac{1}{c_i} v_i \,, \;\; and \;\; W^{(2)} = \sum_{i=1}^{2} \frac{1}{c_i^{\,2}} v_i$$

We use the following algorithm for the minimization of energy functional in (16) as given below:

i) Initialization of ϕ,b and **c.**

ii) Update ϕ to be $\hat{\phi}$ for fixed value of *b* and **c.**

iii) Update *b* to be $\hat{b}$ for fixed value of ϕ and **c.**

iv) Update ***c*** to be $\hat{c}$ for fixed value of ϕ and *b.*

We implement our model by using finite difference scheme with time step δt=0.1, parameters λ_1 =1 and λ_2 =0.001*255^2. We set the scale parameter σ of the Gaussian distribution is 4 and keeping small size of the neighbourhood O_y for the images having more intensity inhomogeneity. Fig.1 shows the result for MRI images. The curve evolution processes has given in the images by showing the initial contours on the images (in the first column), estimated bias field (in the second column) and bias corrected images (in the third column).

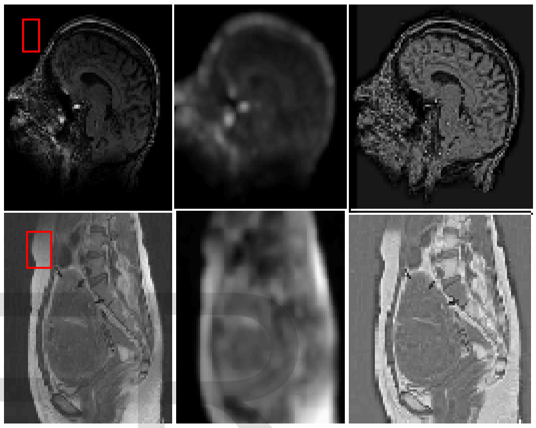

Fig.1: Bias Field Correction of MR Images; Column 1:Orignial image with initial contour; Column 2: Estimated Bias Field; column 3: Bias corrected images

There is an obvious intensity inhomogeneity in the above images. Our method exploits relative intensity information of the images to obtain the estimate unwanted bias field. We correct intensity inhomogeneity by using the estimated value of bias field b and bias corrected image can be obtained dividing the image I by estimated bias field b. To know about the quality of our proposed method, we test our method on medical images with clear intensity inhomogeneity. In Fig.2 the initial contour is shown on the original image in the first column and their corresponding segmentation result, bias field estimation, and bias corrected image are shown in second, third, and fourth column, respectively.

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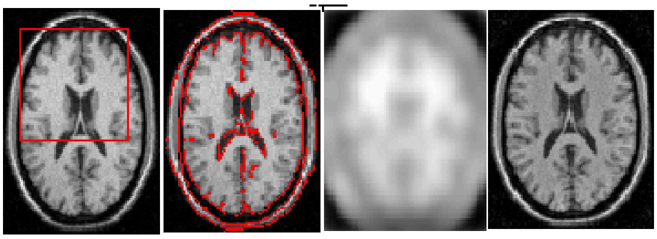

Fig.2.Bias field correction of MR Images.Column1:Initial contour on the original image; Column2: Segmentation result; Column3:Estimated bias field; Column4: Bias corrected image.

It can be seen in Fig.2 the intensities within different region are heterogeneous in the original images and become homogeneous in the bias corrected images. We can judge the result and quality of our proposed model through histogram analysis by observing the histogram of the uncorrected and bias corrected images. The histogram of the original images not showing well-defined and well- separated peak due to the presence of bias field in their intensity distribution. The uncorrected image with initial contour, estimated bias field, bias corrected image are shown in first, second, and third column of row1, histogram of the uncorrected and bias corrected images are shown in the first, second column of row2 in Fig.3.

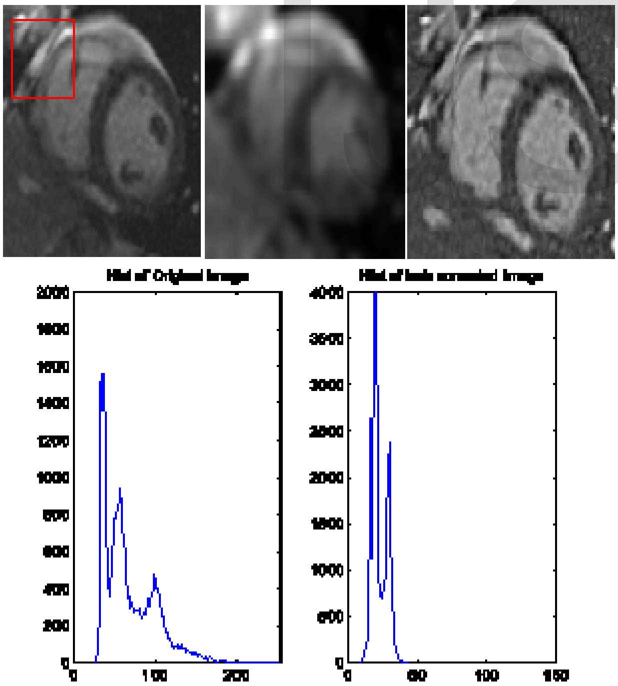

Fig.3:Bias field correction of MR images with Histogram .Row1: Original images; estimated bias field ; bias corrected images; Row2: Histogram of the uncorrected image(left)and bias corrected image(right).

It can be seen in Fig.3 the histogram of the bias corrected images have well-separated peaks shows intensity homogeneity of different region, while histogram of the uncorrected image are not having well-separated peaks due the presence of bias field.

4.4 Comparison with existing model:

Since the Chunming Li model perform well as compared to other active contour models in the presence of bias field, we compare the result of our model with the Chunming Li (CLIC) model in [10]. For this purpose, we are using histogram analysis, because histogram of the bias corrected images will have well- separated peaks showing different homogenous regions in the image domain. Histogram in the second row of Fig.4 is the result of our proposed model having well-defined and well-separated peaks showing different homogenous regions in the bias corrected image, while histogram in the fourth row of Fig.4 is the result of Chunming Li model in [10] do not have well-separated peaks due to the presence bias field.

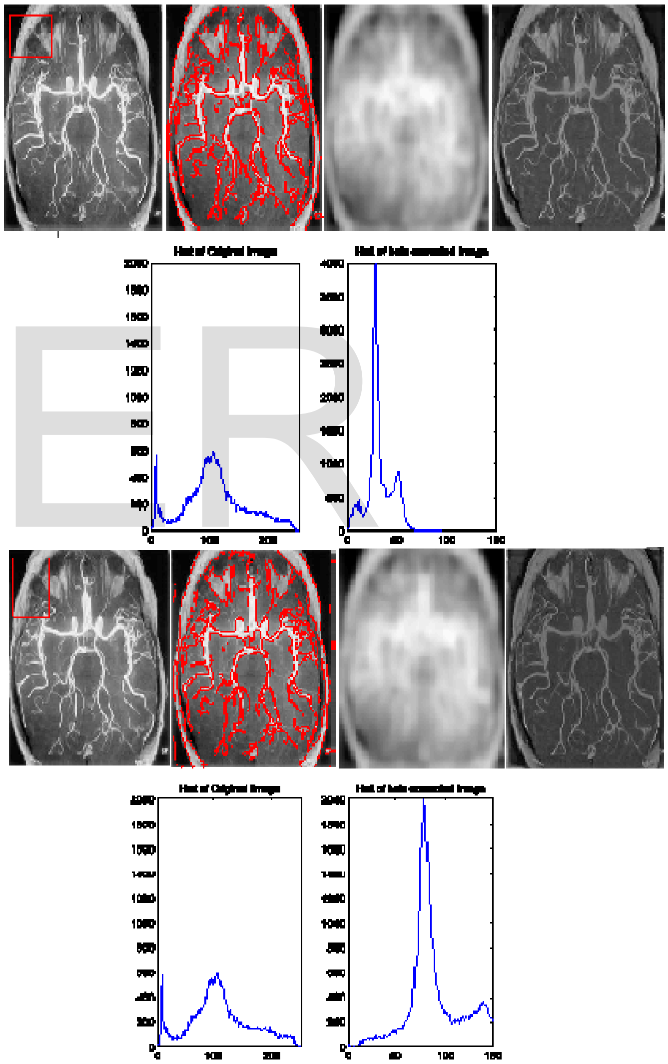

Fig.4. Bias field correction of MR images with Histogram

5. CONCLUSION:

Bias field is a low frequency signal that modifies the intensity value of the images; therefore it is necessary to reduce its value before any image processing techniques. In this paper, we have presented a novel approach for image segmentation with bias field estimation work well

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in the presence of intensity inhomogeneities. We have used local intensity clustering property and define energy functional like coefficient of variation in term of level set function, which uses relative intensity information in small neighbourhood. Minimization of this energy functional result jointly image segmentation with bias field estimation. Empirical results show that our proposed method produce best result in term of efficiency, accuracy and independent of initialization. .

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